"How much further?"
"We've got a hairpin, a thumbnail, and a breathmint to go, according to this map...."
—Erma Bombeck

A

APPENDIX A

MAP SCALE

Maps are always smaller than the environment they represent. The reduction factor is known as the scale of the map. To use maps effectively, you'll need to convert measurements from map units to ground units. As you might expect, an understanding of map scale is central to performing this task. In this appendix, we'll explore the map-scale abilities needed to become a skilled map user.

EXPRESSING SCALE

Map scale is always given in the form, "This little on the map represents that much on the earth's surface." We can state this relation between map and ground distance in two ways—with respect to a linear measurement, or with respect to an areal measurement.

Linear Scale

The relationship between the map and the ground can be expressed in terms of a linear measurement in three ways—with a word statement, a fraction, or a graphic scale.

Word Statement

The most familiar way to express scale is to use a descriptive phrase or **word statement**. We say that there are so many "centimeters to a kilometer" or "inches to the mile." At first it may be confusing to find that one map indicates scale as "one centimeter to the kilometer" and another as "one kilometer to the centimeter." This lack of standardization should cause little trouble, however, since the shorter measure obviously refers to the map while the larger measure refers to the earth.

A more serious problem is the mixture of distance units, such as centimeters and kilometers, in the same word statement. All the time we're using the map, we must consciously keep these different units in mind. The alternative is to use the same unit throughout—to say, for instance, "one centimeter to 200,000 centimeters" rather than "one centimeter to two kilometers." The advantage of this procedure is that the units themselves then become irrelevant and can be put from our minds. Instead of "one centimeter to

200,000 centimeters," the scale might just as well be given as "one inch to 200,000 inches"; it makes no difference in the way we use the map.

Convenient as this method is, though, it introduces an even bigger problem. We have a hard time visualizing what 200,000 centimeters or inches mean in terms of ground distance. It's the same trouble we always have when we try to conceptualize very large numbers. Thus, we're probably better off with our original way of expressing scale. By using larger units of measurement, we eliminate unimaginably huge numbers. The result, regrettably, is that we must learn to think in terms of fractions and multiples of kilometers or miles.

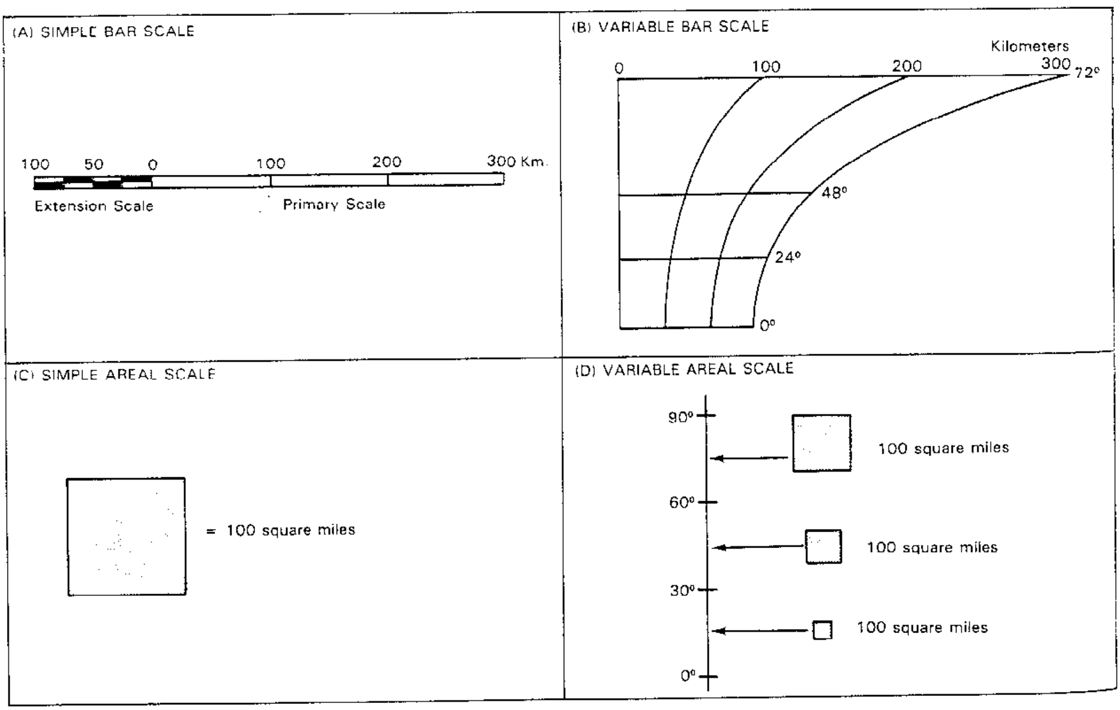
Fraction (Ratio)

A simpler way to describe scale is with a **representative fraction** (**RF** for short). We may also think of this fraction as the **ratio** between map and ground distance. We can write it either as 1/63,360 or 1:63,360. The numerator is always 1 and represents map distance, while the denominator indicates distance on the ground. Both map and ground distance must be given in the same unit of measurement. The advantage of having identical units on the top and bottom of the fraction is that map measurements may be made in centimeters, inches, or whatever unit you choose.

Graphic Scale

A third way to show map-ground relations is to use a **graphic scale**. The simplest of these, called a **bar scale**, looks like a small ruler printed on the map. We usually read this scale from left to right, beginning at 0. Sometimes the scale is extended to the left of the zero point, using smaller markings (**Figure A.1A**). This allows us to determine distance not only in whole units but also in fractions of units.

The marks on the bar scale are arranged so as to provide whole numbers of kilometers or miles of ground distance. This means that the marks won't represent whole numbers of centimeters or inches—there will almost always be some fraction left over. In other words, although



A.1 Several types of graphic scales are found on maps. Shown here are a simple bar scale (A), a variable bar scale (B), a simple areal scale (C), and a variable areal scale (D).

the bar scale *looks* like a ruler, its markings will not coincide with those on your ruler. (Rare exceptions would be a scale of 1:100,000, since at this scale one kilometer on the ground would equal exactly one centimeter on the map, and 1:63,360, since one inch on the map would then be equivalent to one mile on the ground.)

The bar scale has three features which make it especially useful. First, if the map is enlarged or reduced using some method of photocopying, the bar scale changes size in direct proportion to the map. The word statement and representative fraction, on the other hand, lose their meaning when the map changes size. Second, both kilometers and miles can be shown conveniently on the same bar scale. And finally, the bar scale is easy to use when figuring distance on a map, as we see in Chapter 13.

When maps show the whole globe, the scale may vary significantly from one part of the map to another. In such cases, the map maker sometimes replaces the standard bar scale with a **variable graphic scale**. An example of this type of bar scale, taken from a Mercator projection, is given in **Figure A.1B**. Notice that the scale changes systematically in both the north-south and east-west directions. To use such a scale, first decide in what latitude-longitude zone you want to make a distance determination, and then check the scale to see what distance units to use. In effect, you are working with a rubber ruler which can be stretched or shrunk to match the local map scale.

Areal Scale

Although map scales are usually given in linear units such as miles or kilometers, map users are often interested in the size of things in acres or square miles. If the map scale is given in linear units, a conversion to areal units is possible but tends to be a laborious process. Alternatively, the map scale may be given directly in areal units. For example, a word statement might read: one square inch to four square miles (equivalent to a linear scale of 1/126,720).

By far the most common way to show the size of areal units is with the graphic scale. A simple graphic areal scale generally consists of a labeled square or circle of appropriate size (**Figure A.1C**). Variable graphic areal scales consist of a series of squares or circles of different sizes positioned in a latitude-longitude framework (**Figure A.1D**).*

CONVERTING SCALE

If the map maker has been at all conscientious, you will find the scale depicted on the map somewhere. At times, however, it may be in the wrong form to best serve your purpose. Therefore, you may often want to make conversions between a word statement, RF, and graphic scale.

You may, for instance, have a map with a word statement and wish to know the RF. The first thing to remember with any scale conversion is that the ratio is always map distance (numerator) to ground distance (denominator). Suppose that the word statement is three inches to 10 miles. In converting it to an RF, the ratio would be 3"/10 mi. But you can't have a representative fraction with different units on its top and bottom. So you must convert miles to inches—no problem if you just remember that there are 63,360 inches in a miles. Thus:

3"/10 mi. = 3/10 x 63,360 = 3/633,600.

Remember, too, that the numerator of an RF is always 1.** So in this case you will also have to reduce the numerator from 3 to 1 by dividing numerator and denominator by 3.

$$3/633{,}600 = \frac{3/3}{633{,}600/3} = \text{an RF of } 1/211{,}200.$$

Figure A.2 illustrates some other word statement to RF conversion problems.

Sometimes you may find yourself in the opposite situation. You know the RF but want to know how many miles to the inch or inches to the

**The distortion ellipses sometimes drawn on map projections to help viewers understand the degree and pattern of distortion represent a form of areal scale (see Figure C.12 in Appendix C).*

***Hint: In grade school you learned to "reduce fractions" by dividing the numerator and denominator by the same number, since that procedure didn't change the fractional relationship. Thus, you reduce 6/12 to 1/2 by dividing both the top and bottom of the fraction by 6.*

1″ to 3 miles

$$\frac{1''}{3\text{ mi.}} = \frac{1''}{3 \times 63{,}360''} = \frac{1}{190{,}080}$$

5″ to 8 miles

$$\frac{5''}{8\text{ mi.}} = \frac{5''}{8 \times 63{,}360''} = \frac{5''}{506{,}880''} = \frac{5/5}{506{,}880/5} = \frac{1}{101{,}376}$$

15″ to 1 mile

$$\frac{15''}{1\text{ mi.}} = \frac{15''}{1 \times 63{,}360''} = \frac{15/15}{63{,}360/15} = \frac{1}{4{,}224}$$

.2″ to 5 miles

$$\frac{.2''}{5\text{ mi.}} = \frac{.2''}{5 \times 63{,}360''} = \frac{.2''}{316{,}800''} = \frac{.2/.2}{316{,}800/.2} = \frac{1}{1{,}584{,}000}$$

A.2 Converting a word statement to a representative fraction involves manipulating the terms so that the numerator is 1 and both numerator and denominator are in the same units.

mile the map scale represents. If it is miles to the inch that you need, then you merely divide the denominator of the RF by the number of inches in a mile, or 63,360. If it is inches to the mile that you wish, divide the denominator of the RF into 63,360. Similarly, if you want to know kilometers to the centimeter, divide the denominator of the RF by the number of centimeters in a kilometer, or 100,000; if you want centimeters to the kilometer, divide the RF denominator into 100,000.

Some common RF to word statement conversions are provided in **Figure A.3**. Additional conversions can be made indirectly by adding the numbers given in this figure. If the scale of a map is 12 miles to an inch, for example, and you want to know the RF, you can proceed as follows:

	8 miles to an inch is 1:506,880
+	4 miles to an inch is 1:253,440
	12 miles to an inch is 1:760,320

Or, if the RF is 1:79,200 and you need the number of miles to an inch, just reverse the process:

	1:63,360 = 1.00 mile to an inch
+	1:15,840 = .25 mile to an inch
	1:79,200 = 1.25 miles to an inch

Actually, most maps are made at a few common scales. You can therefore save a lot of bother if you figure out beforehand the correspondence between commonly used units of map and ground distance. The best idea is to make your own scale conversion table to which you can refer whenever you need it. This table might take the form shown at the top of Figure A.3.

Sometimes you may want to create a graphic scale from a word statement or representative fraction. There are really two solutions to this problem. You must decide whether you want even units of map distance or of ground distance. Rarely can you have both at once.

Imagine that you have a map with a scale of 3/4 mile to an inch, or 1:47,520. You might want a graphic scale with even inch divisions, since it would then be easy to make measurements on the map with a ruler. You would simply draw a line and divide it into inches, each representing 3/4 mile on the ground (**Figure A.4A**).

Or you might prefer even miles, because that would facilitate ground distance computation. Since an inch equals 3/4 mile, 1/3 inch equals 1/4 mile, and each mile equals 1-1/3 inches. So you would mark off 1-1/3 inch intervals on your graphic scale (**Figure A.4B**).

RF	In. to Miles	Miles to In.	Cm to Km	Km to Cm
1:1,980	32	.03125	50.50	.0198
1:3,960	16	.0625	25.25	.0396
1:7,920	8	.125	12.626	.0792
1:15,840	4	.250	6.313	.1584
1:31,680	2	.5	3.156	.3168
1:63,360	1	1.00	1.578	.6336
1:126,720	.5	2.00	.789	1.2672
1:253,440	.25	4.00	.3946	2.5344
1:506,880	.125	8.00	.197	5.0688
1:1,013,760	.0625	16.00	.0986	10.1376

Worked Examples

(1) How many inches to the mile when RF = 1:24,000?

Solution:

$$24{,}000\,\overline{)\,63{,}360.00\,} = 2.64$$

(2) How many miles to the inch when the RF = 1:24,000?

Solution:

$$63{,}360\,\overline{)\,24{,}000.0000\,} = .3798$$

(1) How many cm to the km. when RF = 24,000?

Solution:

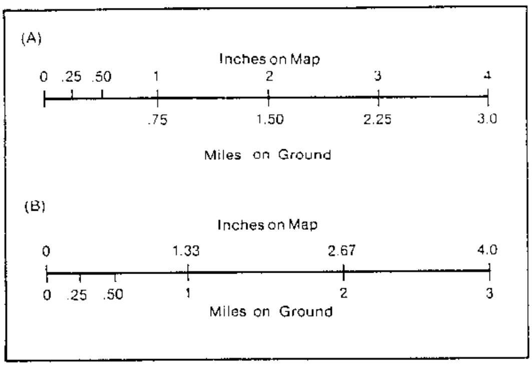
$$24{,}000\,\overline{)\,100{,}000.00\,} = 4.17$$

(2) How many km to the cm. when RF = 24,000?

Solution:

$$100{,}000\,\overline{)\,24{,}000.00\,} = .24$$

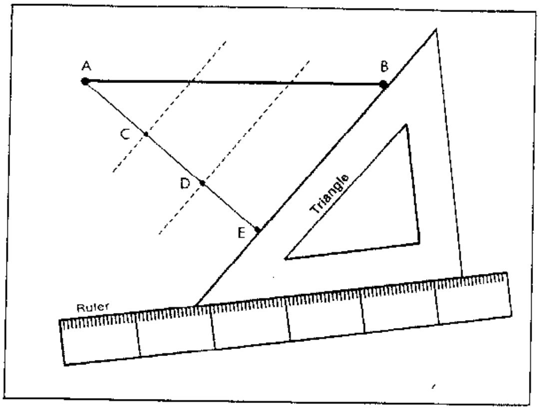
A.3 Converting a representative fraction to a word statement.



A.4 A graphic scale can be divided so that either the map units are whole numbers (A) or the ground units are whole numbers (B). (Caution: These bar graphs are not drawn to scale.)

You'll have no trouble doing this if your ruler is marked with 1/3 inch increments. But not all rulers are. When you make graphic scales, you will often run into this problem of trying to divide a line into segments which aren't found on your ruler. The thing to do then is to fall back on an old trick of plane geometry concerning the relation of parallel lines.

Let's use our previous example and assume that you want to make a graphic scale with 1-1/3 inches to a mile. First you must decide how long to make the graphic scale. Because it would be convenient if your scale showed an even number of inches as well as miles, keep adding 1-1/3 inches until you come up with an even number. (If you don't arrive at an even number within a reasonable amount of time, abandon this method and



A.5 A line can be divided into equal segments using a triangle and straightedge.

choose an arbitrary length for your graphic scale.) If you add 1-1/3" + 1-1/3" + 1-1/3", you get an even four inches, which represents three miles. So four inches is a good length for your graphic scale. Draw a four-inch line, and label it AB (**Figure A.5**).

Now draw a second line at an acute angle from your first line. It doesn't matter exactly where you draw it, but an angle of less than 75 degrees will be most convenient. Next, starting at point A, mark off three equal divisions on this second line, say at one-inch intervals. Label these points C, D, and E. Then connect points B and E with a line.

Now place a right triangle along line BE and a ruler along the bottom of the triangle, as shown in Figure A.5. Slide the triangle along the ruler to point D, and draw a line from D up to line AB. Slide the triangle to point E, and draw another line to AB. What you have done is to draw a series of parallel lines which neatly divide your graphic scale into thirds. Since your original line was four inches long, each of those thirds is 1-1/3 inches.

DETERMINING SCALE

It's a good feeling to know that, no matter what sort of map scale you encounter, you can change it to the type of scale you want. But what if you come across a map with no scale depicted at all? This happens more often than you might expect. You may want to know the scale of an air photo, for instance, or of a photocopied portion of a map on which no scale has been shown.

You can figure out the scale on your own if you know the ground distance between any two points on your map. Then you just measure the distance between those same two points on the map. The ratio of map to ground distance will be the map's scale.

How, though, do you find the ground distance you need? There are several ways. One method is to use some terrestrial feature whose length is known.

Determining Scale With a Terrestrial Feature

Some features have standard dimensions. If you can identify one of these on your map, and if you can be sure that it's shown in proportional size, then you can easily figure out the map scale. A regulation U.S. football field, for example, can be assumed to be 100 yards long. If the map distance of the field is .5 inch, then .5 inch on the map represents 100 yards on the ground. To determine the map scale, we merely convert yards to inches and reduce the numerator to 1:

$$\frac{.5''}{100 \text{ yd.}} = \frac{.5''}{100 \times 36''} = \frac{.5''}{3{,}600''} =$$

$$\frac{.5/.5}{3{,}600/.5} = \text{an RF of } 1/7{,}200.$$

Determining Scale With Reference Material

If you can't find a feature of standard dimensions on your map, you can still determine scale if you turn to other reference material, such as gazetteers, distance logs, or atlases. From these sources, you should be able to find out the distance of something on your map—the length of a lake, say, or the dimensions of a political boundary, or the distance between two prominent features such as cities. With this information in hand, you can compute the map scale as was done above when we used a feature of standard dimensions.

You can follow the same procedure to determine the scale of **global maps**. Here you make use of the fact that the earth's circumference is approximately 25,000 statute miles (the actual figure is 24,901.92 statute miles). If, for example, a world map extends the length of an eight-inch textbook page, you find the map's scale as follows:

$$\frac{8'' \text{ page (map distance)}}{25{,}000 \text{ miles (ground distance)}} =$$

$$\frac{8''}{25{,}000 \text{ miles} \times 63{,}360''} =$$

$$\frac{8''}{1{,}584{,}000{,}000''} =$$

$$\frac{8/8}{1{,}584{,}000{,}000/8} =$$

an RF of 1/198,000,000 or a word statement of 198,000,000 ÷ 63,360 = 1 inch to 3,125 miles.

Determining Scale With Latitude-Longitude Lines

It isn't always convenient or even possible to find a feature such as a football field or a lake of known length on your map. But on many maps, especially those of small scale, latitude-longitude lines are shown. Thus, you can determine scale by finding the ground distance between these lines.

Finding the ground distance between latitude lines is quite simple, since a degree of latitude varies only slightly from pole to equator. The first degree of latitude north or south of the equator extends 110.567 kilometers (68.703 statute miles), while a degree of latitude adjacent to the North or South Pole covers 111.699 kilometers (69.407 miles). Thus, the variation in a degree of latitude from equator to pole is only 1.132 km. or .704 mile. This discrepancy is so small that for many purposes it can be ignored. We say that a degree of latitude is equivalent to a degree of longitude at the equator, which is **69.172 miles**, regardless of where on earth the degree of latitude is found. (If you need a more precise measure of the length of a degree of latitude, refer to Table E.6 in Appendix E.)

In order to use this information in determining map scale, you must be able to find at least two parallels on your map. Let's say you find two latitude lines separated by an increment of two degrees and a map distance of five inches. To find the ground distance between these parallels, you multiply the number of degrees separating them by the length of a degree. Thus, 2° x 69.172 = 138.344 statute miles. Now you can form a relation between map distance and ground distance, yielding 5"/138.344 miles = 5"/138.344 x 63,360" = 5"/8,765,475" = an RF of 1/1,753,095 or a word statement of 1,753,095 ÷ 63,360 = 28 miles to the inch.

Unfortunately, two parallels are not always shown on a map. You can still compute the map scale, however, if two meridians are shown. The trouble is that longitude lines are not as consistent as latitude lines. Because meridians converge at the poles, a degree of longitude varies from 111.321 kilometers (69.172 miles) along the equator to 0 kilometers at either pole (see Table E.7 in Appendix E). This means that the distance between longitude lines depends on the mapped region's latitude. Luckily, there's a simple functional relationship between latitude and longitude: Longitude changes as the cosine of the latitude. (You'll find the cosine you need in Table E.5 in Appendix E.) To find the length of a degree of longitude, you multiply the length of a degree of latitude (111.321 kilometers or 69.172 miles) by the cosine of the latitude. This relationship can be written symbolically as

$$\text{Longitude} = \cos(\text{latitude}) \times 111.321 \text{ km.}$$

or

$$\text{Longitude} = \cos(\text{latitude}) \times 69.172 \text{ miles.}$$

Now how can you use this equation to determine map scale? Although the procedure may at first seem complex, it is actually straightforward if viewed as a series of simple steps. You begin by measuring the map distance between two longitude lines. Suppose you find that lines spanning 1/2° of longitude on the map are 10 inches apart. To determine ground distance between these two longitude lines, first find the latitude of the region on the map. Then check the table of cosines in Appendix E to find the length of a degree of longitude at this latitude. Suppose the latitude is 45°N. In this case,

$$\begin{aligned} 1° \text{ longitude} &= \cos(45°) \times 69.172 \\ &= .7071 \times 69.172 \\ &= 48.9 \text{ miles} \end{aligned}$$

Thus, you know that one degree of longitude at 45°N covers a ground distance of 48.9 miles. But since the measured map distance spanned 1/2° rather than 1° of longitude, the ground distance in this problem would be half the computed figure, or .5 x 48.9 = 24.45 miles.

You now have all the information necessary to find the map scale. Simply form a ratio between map and ground distance, obtaining 10"/24.45 miles = 10"/24.45 x 63,360" = 10"/1,549,152" = an RF of 1/154,915 or a word statement of 154,915 ÷ 63,360 = approximately 2.5 miles to the inch.

Determining Scale by Comparison

If you aren't able to determine the scale with any of the above methods, you might try comparing your scaleless map with other maps of the same region which do have their scales identified. It should be possible to place any map along a continuum between maps of known scales. This procedure is only approximate, but sometimes a rough idea of a map's scale is all you need.

SCALE PROBLEMS

We often compare maps according to the relative size of their scales. **Small-scale** maps result when map distance is small relative to ground distance—that is, when the scale ratio is small; on **large-scale** maps, the map-ground ratio is large.

Since map distance is always stated in the numerator of the RF as 1, it follows that the smaller the denominator (the closer to a 1 to 1 ratio), the larger the scale will be. Thus, a map scale of 1:20,000 is twice as large as a scale of 1:40,000. If that sounds backwards, remember that the RF is a fraction, and 1/2, after all, is a larger proportion (or piece of the pie) than 1/4.

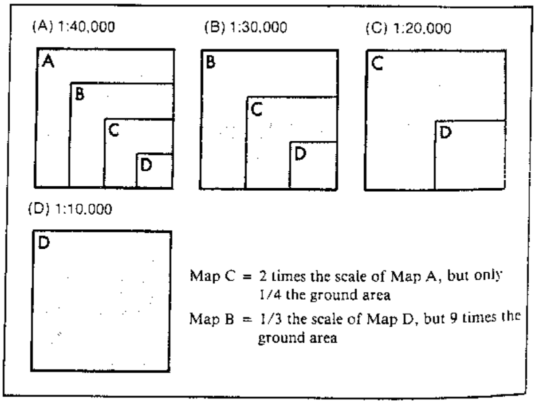
Common as it is to classify maps by their scales, there is no general agreement as to where the class limits should be set. If we sort maps into two groups—large and small scale—then 1:1,000,000 would be a likely dividing point between the two. Atlas, textbook, and wall maps of continental coverage would then fall into the small-scale group, while topographic, cadastral, and other sheet maps would be in the large-scale class. If a more detailed three-way grouping is used, maps with scales of 1:1,000,000 and smaller (16 or more miles to the inch) would probably be classed as small-scale and those of 1:250,000 and larger (four or less miles to the inch) as large-scale. Maps ranging in scale between these extremes would then be referred to as medium-scale. Any such classification, of course, is arbitrary and shouldn't be given meaning beyond the organizational convenience it provides.

It's often useful to know the relation between map scale, map size, and ground coverage. You may, for instance, want to know the relationship among ground areas shown on four maps of equal size but of different scales. The rule, as **Figure A.6** shows, is that ground area changes as the square of the linear distance. Although Map C is two times the scale of Map A, it depicts a ground area only 1/4 as great ($2^2 = 4$). Conversely, Map B is 1/3 the scale of Map D, yet it shows a ground area which is nine times as large. In other words, a map's ground coverage varies inversely with the square of any scale change. Or, stated symbolically:

$$GC = 1/\text{scale change}^2.$$

If a map's scale is increased by a factor of four, then, the area that can be shown on a map of the same size decreases to 1/16 the original value ($GC = 1/4^2 = 1/16$).

Let's see how we can use this information.



A.6 The ground coverage of a map changes in inverse proportion to the square of the map scale. Cutting the map scale in half increases the ground coverage by a factor of four.

Problem 1. Using a photocopy machine, you reduce a map with a scale of 1:90,000 to 45 percent its original size. What will be the scale of the reduced map? Remember that, although the map distance will be reduced to 45 percent, it will still represent the same ground distance.

Answer:

$$\frac{1}{90{,}000} \times \frac{.45}{1} =$$

$$\frac{.45}{90{,}000} = \frac{.45/.45}{90{,}000/.45} = 1/200{,}000$$

Problem 2. On one map, with a scale of 1:100,000, the distance between two cities is .7 cm. On a second map, the distance between the same two points is .5 cm. What is the scale of the second map?

The solution involves equating distance on the first map to distance on the second. Answer:

$$(.5 \text{ cm.})\,(X) = .7 \text{ cm.}\,(100{,}000)$$
$$.5X = 70{,}000$$
$$X = 70{,}000/.5$$
$$X = 140{,}000$$

Problem 3. On a map with an RF of 1:100,000, there is a rectangular reservoir measuring 12 by 5 centimeters. What is the area of the reservoir in square kilometers?

The key is to recall that there are 100,000 centimeters in a kilometer. Therefore, the map scale is one cm. to the km.

Answer:

$$12 \text{ cm.} \times 5 \text{ cm.} = 60 \text{ cm.}^2$$

Since 1 cm.^2 on the map = 1 km.^2 on the ground, 60 cm.^2 = 60 km.^2.

Problem 4. On the first of two maps, a square parcel of land measures 2.7 cm. by 2.7 cm. On the second map, whose RF is 1:30,000, the same parcel of land occupies 1/9 as much area as on the first map. What is the RF of the first map, and what is the area of the parcel of land in square meters?

The best way to solve this problem is to begin with a picture. **Figure A.7** shows that the second map is of smaller scale than the first map, since it covers the same ground area with only 1/9 the map area. Remembering our scale changing rule (GC = 1/scale change^2), we know that area changes as the square. Thus, it would take a linear change of 1/3 to produce an area change of 1/9, since $1/3^2 = 1/9$. This means that 2.7 cm. on the first map is equivalent to 1/3 x 2.7 = .9 cm. on the second map.

In other words, a given distance on the first map (X_1) represents 1/3 times the same distance on the second map (X_2). Or, written symbolically, $X_1 = 1/3 \times X_2$. Since we know that the RF of the second map is 1/30,000, then $X_2 = 1 \times 30{,}000 \times 30{,}000$. To find X_1, we merely multiply X_2 by 1/3. So the RF of the first map is 1/30,000 x 1/3 = 1/10,000.

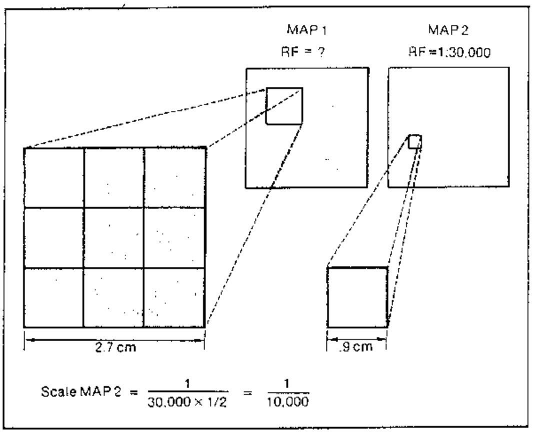
The area of the parcel of land can be determined using either .9 cm. at the scale of 1:30,000 or 2.7 cm. at the scale of 1:10,000. Using the latter RF and letting S represent the length of a side of the land parcel, we get:

$$S = 2.7 \text{ cm. (map distance)} \times 10{,}000 \text{ cm. (ground distance)} = 27{,}000 \text{ cm.}$$

We then transfer this value into the formula for obtaining the area of a square:

$$\text{Area} = S^2$$
$$= (27{,}000 \text{ cm.})^2$$
$$= 729{,}000{,}000 \text{ square cm.}$$

Since there are 10,000 square centimeters in a square meter, 729,000,000 square centimeters = 72,900 square meters.



A.7 A graphic representation of Scale Problem 4 (see text).